Introduction to the numerical stability method used in the analysis

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1 Stability of the convection term

In the Navier-Stokes equations the convection is non-linear. Regions of stability (inside the "circles") for the Adams-Bashforth (AB) family is shown in Figure 1.

For stability the time step restriction (Δt) is scaled to get into the positive region

$$\Delta t \frac{u}{h} \leq \Delta t_{cr} \frac{u}{h} = 0.723 \quad (1)$$

$$h \to 0 \qquad \Delta t_{cr} \to 0 \quad (2)$$

There is a linear relationship between the element size (h) and the critical time step (Δt_{cr}). For example, if the element size decreases by a factor of 2 the critical time step also decreases by a factor of 2.

AB3 is implemented as follows:

$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{h}\left(\frac{23}{12}Cu^n - \frac{16}{12}Cu^{n-1} + \frac{5}{12}Cu^{n-2}\right) \quad (3)$$

the stability criteria can be expressed as

$$\Delta t \leq \Delta t_{cr} = \min\left(\frac{h'}{u'}\right) \cdot 0.723 \cdot \mathrm{SF} \quad (4)$$

where SF is a safety factor (e.g., SF=0.9) and

$$h'_b = |X_b - X_a| = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2} \quad (5)$$

$$u'_b = \frac{|U_b \cdot (X_b - X_a)|}{|X_b - X_a|} = \frac{|(u_x)_b(x_b - x_a) + (u_y)_b(y_b - y_a)|}{\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}} \quad (6)$$

where $X = (x, y)$, $U = (u_x, u_y)$.

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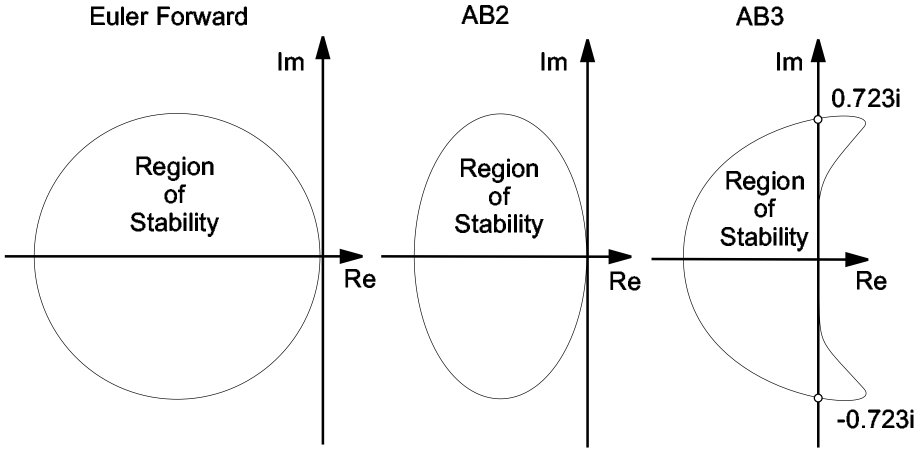
Figure 1: Adams-Bashforth family for stability



These calculations should be performed for each node and for each neighboring node to find the minimum for the time step restriction. Since the velocities are zero on the boundaries, Δt_{cr} should be not calculated for these nodes. Figure 2 gives an example of the nodes referred to in the equations above.

Figure 2: Node sketch for stability calculations